

a press of other work precluded them from comparison with the instrument (standard) at Kew Observatory. After inspection of the observations, you may be good enough to consider the desirability of sending me a third actinometer of the same kind, due comparison being first made at Kew, whereby the relation between the instrument at the Observatory (Kew) and those sent out to me by the Royal Society may be established.

XV. "On the Radiation of Heat from the Moon.—No. II." By the EARL OF ROSSE, F.R.S. Received June 14, 1870.

In a former communication to the Royal Society I gave a short account of some experiments on the radiation of heat from the moon, made with the three-foot reflector at Parsonstown, during the season of 1868–1869. I then showed:

1st. That the moon's heat can be detected with certainty at any time between the first and last quarter, and that, as far as could be ascertained from so imperfect a series of observations, the increase and decrease of her heat, with her phases, seems to be proportional to the increase and decrease of her light, as deduced by calculation*.

2ndly. That a much smaller percentage of lunar than of solar rays is transmitted by a plate of glass, and we therefore infer that a large portion of the rays of high refrangibility, which reach the moon from the sun, do not at once leave the moon's surface, but are first absorbed, raise the temperature of the surface, and afterwards leave it as heat-rays of low refrangibility.

3rdly. That, neglecting the effect of want of transparency in our atmosphere, and assuming, in the absence of any definite information on the subject, that the radiating-power of the moon's surface is equal to that of a blackened tin vessel filled with water, the lunar surface passes through a range of 500° F. of temperature; consequently the actual range is probably considerably more.

4thly. The proportion between the intensity of sunlight and moonlight, and between the heat which comes from the sun and from the moon, as deduced from those observations, agreed as nearly as could be expected with the values found by independent methods, and for this reason might be considered the more reliable.

During the past season these observations have been continued, but much time has been spent in trying various modifications of the apparatus, and a satisfactory comparison of observations made on different nights, under different circumstances, has been impossible; however, by more numerous and more complete experiments, made alternately with and without an inter-

* See the Proceedings of the Royal Society, No. 112, 1869, page 439.

posed plate of glass, the second conclusion arrived at during the previous season has been to a great extent confirmed.

The following Table gives the values found for the percentage of the moon's heat which passes through glass :—

Date of observation.	Distance of moon from opposition.	Altitude of the moon.	Percentage of moon's heat transmitted by glass.	
April 15th, 1870	0	0		
	5	{ 15	13·3	I.
		{ 20½	15·5	II.
		{ 24	16·6	III.
April 16th	15	{ 15	14·5	I.
		{ 24	14·6	II.
April 17th	31	{ 19½	10·0	I.
		{ 20	10·0	II.
March 13th	50	50	7·1	
February 10th ...	66	44	8·4	
February 9th	77	32	9·3	
April 9th	81	{ 44	11·0	I.
		{ 16	11·0	II.
April 8th	93	30	12·0	
March 8th	109	27	13·0	

Mean = 11·88.

The same plate of glass which was used in I. and II. on April 15th, and the experiments on the two following nights, was tested for the solar rays, and the following values of the percentage of heat transmitted were obtained :—

April 15th.....	86·2
	86·6
April 18th.....	89·3
	84·3
	87·1
Mean on April 18th.....	86·8

The piece of glass used on the other occasions, instead of being placed at six or eight inches from the pile, was laid against the end of the protecting cone, or about half an inch from the face of the pile. When it was placed in this position and tested for solar rays, an increase of deviation in the proportion of 1·1 to 1 was obtained, owing to the “bottling up” of the sun's rays as in an ordinary greenhouse, and the keeping off of currents of air.

It seems therefore to be clearly proved that there is a remarkable difference between the sun's and the moon's heat in regard to their power of passing through glass. The amount transmitted varies from night to night, and in the later observations the value was generally larger than in the earlier ones. Possibly this may have arisen from the formation of a

slight and imperceptible film of moisture on the surface of the glass, which was much more unlikely to form during the much shorter period* of exposure to the night air in the later observations.

The experiment made during the previous season to determine the ratio between the heating-power of the moon and of the sun was repeated with more care, and the value found, taking what appeared to be the most probable mean heating-power of full moon, as determined on various nights, was

$$\frac{\text{Sun's total heat}}{\text{Moon's total heat}} = 82600.$$

Taking† the percentage of light transmitted by glass = 92

Do. do. of sun's heat = 87

Do. do. of moon's heat = 12

Do. do. of heat from a body at 180° F. = 1·6

If $\frac{0}{0+l}$ and $\frac{l}{0+l}$ represent respectively the percentage of dark and luminous rays present in the moon's radiant heat, and $\frac{0'}{0'+l'}$ and $\frac{l'}{0'+l'}$ the corresponding quantities for the sun's radiant heat, we have

$$\frac{0 \times \cdot 016 + l \times \cdot 92}{l + 0} = \cdot 12,$$

and

$$\frac{0' \times \cdot 016 + l' \times \cdot 92}{l' + 0'} = \cdot 87;$$

$$\therefore \frac{l'}{l} = \frac{0+l}{0'+l'} \times \frac{854}{104} = 82600 \times \frac{854}{104} = 678300.$$

In all the foregoing experiments on lunar radiation the quantity measured by the thermopile was the difference between the radiation from the circle of sky containing the moon's disk and that from a circle of sky of equal diameter not containing the moon's disk; we have obtained no information in reference to the absolute temperature of either the moon or the sky.

The following experiment was therefore made with the view of trying to connect the radiation of the sky with that of a body of known temperature, the deviation due to each degree (Fahrenheit) difference of temperature between a blackened tin vessel containing hot water and subtending a given angle at the pile and a similar vessel containing colder water was first ascertained; then a similar determination of that due to the difference of radiation from one of these vessels, and from a portion of sky of equal diameter, was made. The following was the result:—

* About 12 minutes in place of 30 to 60 minutes.

† All these values, except the first, were determined by experiment for the specimen of glass employed.

	Altitude of part of sky examined.	Calculated difference of tem- perature.	Tempera- ture of tin vessel.	Apparent tempera- ture of the sky.	Remarks.
April 16th	49	23·9	55·5	31·6	Sky hazy. } Sky apparently black and transparent; occasional light clouds.
April 20th	49	32·7	49	16·3	
"	23½	28·3	51	22·7	
"	50	28·3	50·5	22·2	
"	64	30·1	47	16·9	
"	64	26·2	44	17·8	

If the temperature of space be really as low as is supposed, this result seems to indicate considerable opacity of our atmosphere for heat-rays of low refrangibility.

The ever varying transparency of our atmosphere has been found to be a very serious obstacle; but the much greater steadiness of the needle during the later experiments (the mean error of the last few nights' observations having been from two to three and a half per cent. only of the whole deviation*) encourages us with the hope that, by taking advantage of favourable moments, and measuring the moon's light simultaneously with her heat, more accurate information on this subject may soon be acquired.

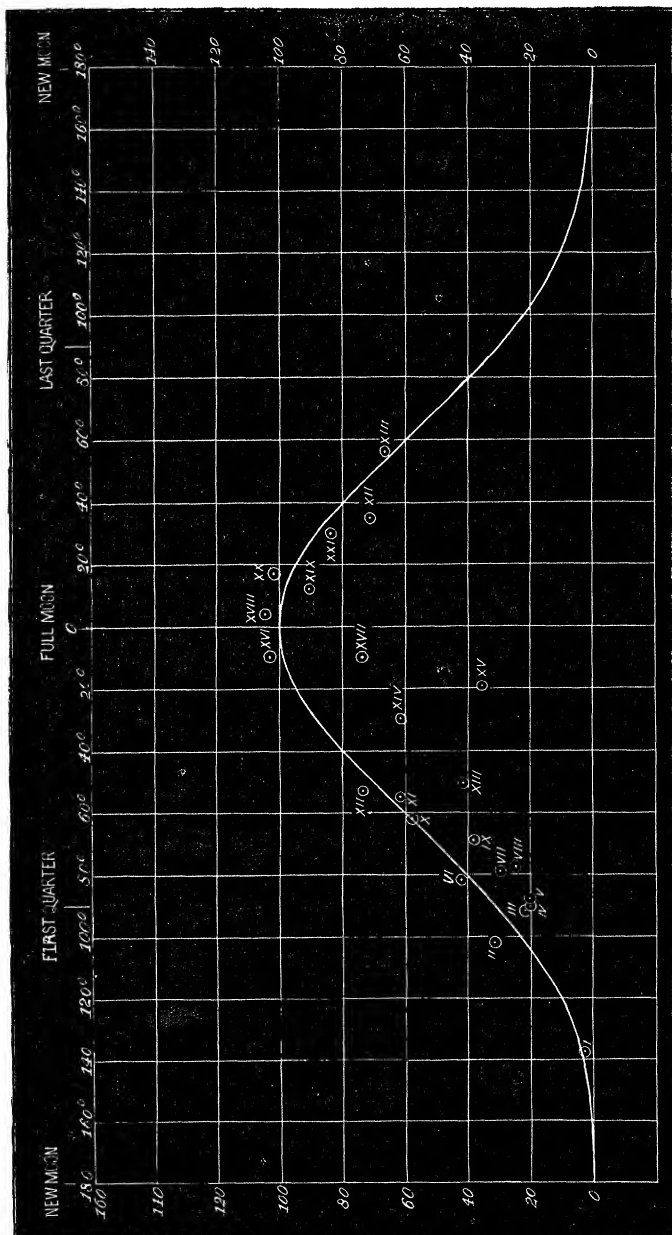
The observations were examined with the view of ascertaining how far the heating-power of the moon's rays varies with her altitude. Owing to the interference of clouds, and the limited range of altitude within which the observations were made, it is hardly worth while to give the results in detail; however, I may just say that the heating-power of the moon's rays appears to diminish with her altitude only about one-third as fast as the intensity of the solar chemical rays, as ascertained by Roscoe and Thorpe.

An attempt was made to ascertain, by comparing two measurements of the moon's light at different altitudes with two corresponding measurements of her heat, whether our atmosphere intercepts the *heat*-rays to a greater extent than the *luminous* rays. It was found that while the light was diminished with the altitude in the proportion of about 3 to 1, the heat was diminished in the proportion of about 5 to 1. In consequence, however, of much of the moon's light and heat being intercepted by hazy clouds, or *condensed* vapour, at the lower altitude, the experiment was inconclusive as to the effect of a *transparent* atmosphere on the dark rays of heat.

The accompanying diagram shows the proportion between the amount of lunar heat found on various nights at various ages of the moon. There appears to be a general accordance between the variation of her radiant heat with her phase and the corresponding amount of her light as deduced by calculation.

* During the experiments of the previous season the mean error varied between 27 per cent. and 85 per cent. or more.

As far as we can judge from so few and imperfect experiments, the maximum of heat seems to be a little after full moon.



Subjoined is a Table giving the dates of the various observations, with the reference numbers corresponding to those on the diagram, and with remarks on the state of the sky.

Number in diagram.	Date of observation.	Remarks.
I.	April 4th	
II.	January 8th	No mention of cloud.
III.	April 8th	
IV.	January 9th	No mention of cloud.
V.	March 8th	Extremely clear sky.
VI.	April 9th	No mention of cloud. [night by a halo.
VII.	January 10th ...	Sky not good; thin hazy clouds, followed later in the
VIII.	February 9th.....	
IX.	January 11th.....	Much wind.
X.	February 10th ...	No mention of clouds.
XI.	January 12th.....	Occasional small clouds, and rather hazy.
XII.	November 19th ...	Clouds producing prismatic colours round the moon.
XIII.	March 13th	
XIV.	April 13th	Sky not good; fleecy clouds. [clouds.
XV.	April 14th	Bad night; stopped after 10 minutes, in consequence of
XVI.	April 15th ..	Sky very clear.
XVII.	January 16th.....	
XVIII.	September 20th ...	Occasional clouds.
XIX.	February 16th ...	Sky hazy at sunset; occasional clouds. [night.
XX.	April 16th	Sky apparently not quite so clear as on the preceding
XXI.	April 17th	
XXII.	November 22nd...	Fog and white frost, afterwards drift.
XXIII.	November 23rd...	No remark about cloud.

XVI. "On Linear Differential Equations."—No. III. By W. H. L. RUSSELL, F.R.S. Received June 11, 1870.

The integrals obtained in my last paper on this subject were deduced by the same process which afforded the determinants in the first paper. It is obvious that these integrals could be found by a more direct investigation. This is what I am now going to attempt. It will be found moreover that the present method will have the advantage of clearing away the ambiguities arising from the existence of common factors in the algebraical coefficient of the highest differential, and the denominator of the exponential in the solution. It will also be found to lead us to certain ulterior results.

Let us take the differential equation

$$(\alpha + \beta x) \frac{d^3 y}{dx^3} + (\alpha' + \beta' x + \gamma' x^2) \frac{d^2 y}{dx^2} + (\alpha'' + \beta'' x + \gamma'' x^2) \frac{dy}{dx} + (\alpha''' + \beta''' x + \gamma''' x^2) y = 0.$$

Let us now put in this equation

$$y = E(x) e^{\int dx (\mu + vx)}.$$

We shall easily see that it is impossible for the exponential to contain

NEW MOON

FIRST QUARTER

FULL MOON

LAST QUARTER

NEW MOON

